CENF

Common European Numeracy Framework

Warning!

This presentation is on

Numeracy

Gaps
Deficiencies
Diagnoses
Anxiety
Right / wrong

erasmus+

Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT

bfi

UNIVERSITAT DE BARCELONA

UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

Numeracy

CENF
Common European Numeracy Framework

Numeracy is an human activity
Numeracy is functional and highly practical

Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT

bfi

UNIVERSITAT DE BARCELONA

UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

erasmus+

CENF
Common European Numeracy Framework

Common European Numeracy Framework

- Two serious challenges
- Context: Developing CENF in Erasmus+
- Theoretical underpinning / choices
- Overview of the construct
- Awareness, cooperation and dissemination

Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT

bfi

UNIVERSITAT DE BARCELONA

UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

erasmus+

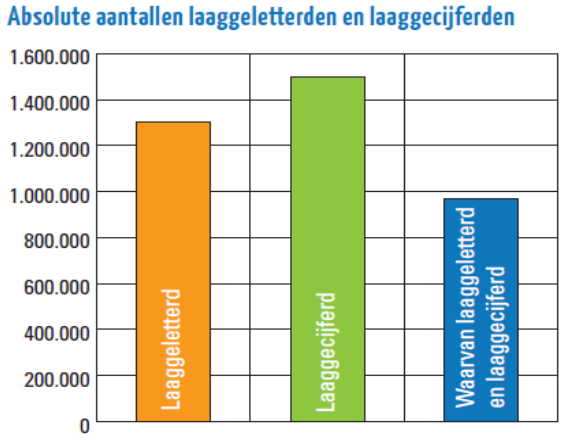
Two serious challenges

CENF
Common European Numeracy Framework

- Awareness of the importance of numeracy for personal empowerment and for societal and economic development.

- "Too many European citizens lack the necessary numeracy competencies to participate autonomously and effectively in our technologized and number-drenched society and consequently many citizens are overlooked for certain jobs and have problems in their daily life, dealing with the abundance of number-related issues (OECD, UNESCO, EU)".

- The amounts of citizen with low numeracy across Europe.



Op basis van de beroepsbevolking in 2012: 10.992000 (CBS).
Bron: Buisman e.a., 2013.

Left to right:
- Low literacy
- Low numeracy
- Both

erasmus+

Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT

bfi

UNIVERSITAT DE BARCELONA

UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

Developing the CENF in Erasmus+

- Policy input
  - The **2019 European Numeracy Survey** across Europe (UL, Ireland)
  - Personal en professional networks around adult numeracy education
- Theoretical input
  - Systematic Literature Review on Numeracy (UB, Spain)
  - Existing supranational frameworks
    - PIAAC (1st and 2nd cycle) / PISA 2015, 2021 (OECD)
    - Principles and Standards (NCTM, USA)
    - ACARA, Australia
- Empirical Input
  - Professional development modules and trials (BFI, Austria)

CENF
Common European Numeracy Framework

erasmus+
Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT
bfi
UNIVERSITAT DE BARCELONA
UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

Theoretical underpinning

CENF
Common European Numeracy Framework

- Numeracy is basic arithmetic (1950-1975)
- Numeracy is subset of mathematics (1975-2000)
- Numeracy is subset of literacy, namely

  numerical literacy (UNESCO)
- Numeracy must be seen as **numerate behaviour** (1990 - …)
  - ALL, IALS, PIAAC assessment frameworks
- Numeracy as **social practice** (2000 - …)

Co-funded by the Erasmus+ Programme of the European Union

PIAAC assessment frameworks

CENF
Common European Numeracy Framework

Coverage: Facets of Numerate behavior…

Involves managing a situation or solving a problem…

1. **in a real context…**
   everyday life, work, societal, further learning
2. **by responding…**

10% - identify, locate or access
40% - act upon, use: order, count, estimate, compute,
30% - interpret, evaluate measure, model
20% - communicate

3. **to mathematical content/ information/ ideas…**

30% - quantity & number
20% - dimension & shape
30% - patterns, relationships, change
20% - data & chance

4. **represented in multiple ways:**
   - objects & pictures
   - numbers & mathematical symbols, formulae
   - diagrams & maps, graphs, tables
   - texts
   - technology-based displays

erasmus+

Co-funded by the
Erasmus+ Programme
of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT

bfi

UNIVERSITAT DE BARCELONA

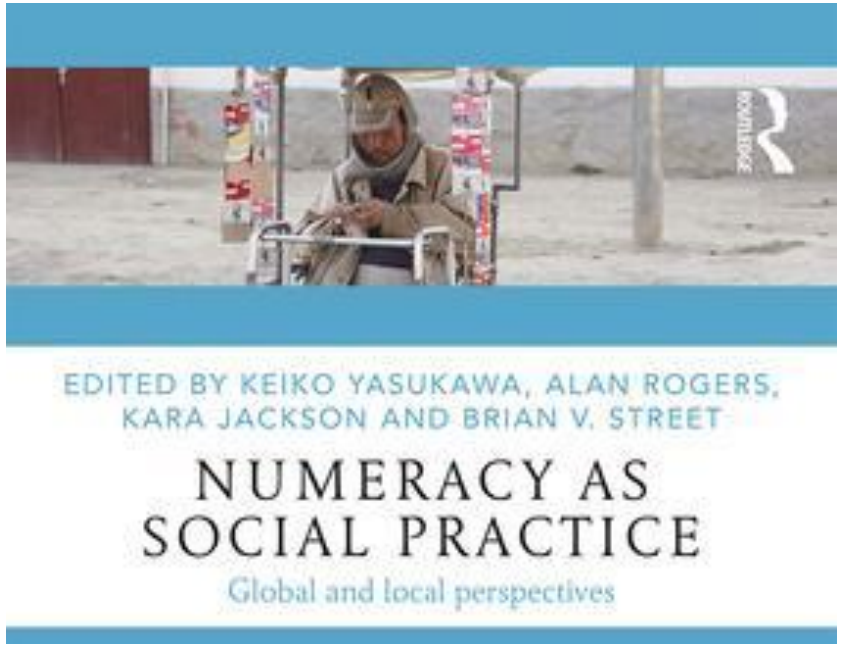
UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

Numeracy as social practice (NSP)

CENF
Common European Numeracy Framework

A social practice view of numeracy not only takes into account the different contexts in which numeracy is practised, such as school, college, work and home, but also how people's life and histories, goals, values and attitudes will influence the way they carry out numeracy.

(See Oughton, 2013)



- Research-informed by
  - Situated cognition
  - Cultural-historical activity theory (CHAT)
  - Literacy as social practice (LSP)
  - Ethnomathematics

(See Yasukawa et al., 2018)

erasmus+

Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT

bfi

UNIVERSITAT DE BARCELONA

UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

Implications for a framework

CENF
Common European Numeracy Framework

- Acknowledging Numeracy as a social practice
- Levels
  - to define progress as a result of educational interventions
  - to categorize job Implies describing behaviour in a “valued system”
  - to categorize test items
  - to categorize psychological scales
  - ….
- Multidimensional (cognitive and psychological) approach
  - Implies multidimensional individual profiles

erasmus+

Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT

bfi

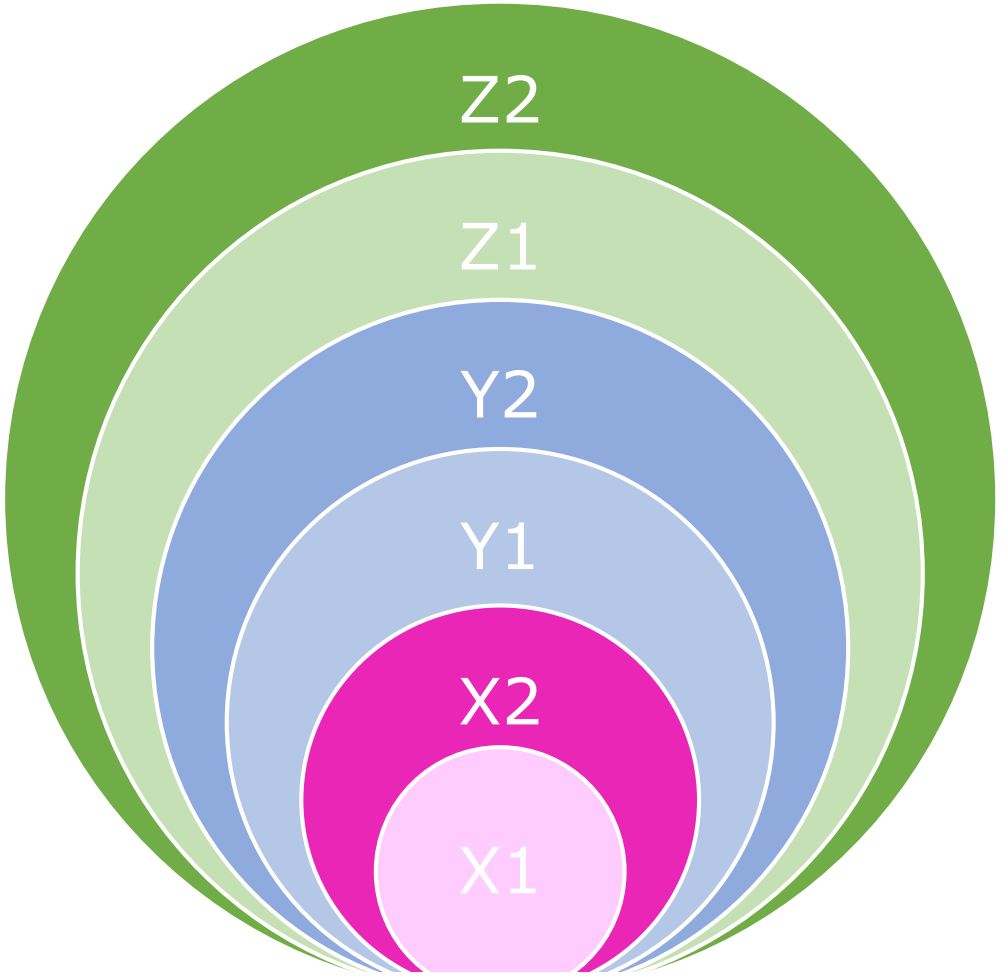
UNIVERSITAT DE BARCELONA

UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

Overall levels (= categories ≠ tresholds)

CENF
Common European Numeracy Framework

Z Professional use	Z Specialized societal and work situations	Z Proficient user
Y Citizen use	Y Societal and regular work situations	Y Advanced User
X Personal use	X Daily-life situations	X Starting user



Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT

bfi

UNIVERSITAT DE BARCELONA

UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

CENF
Common European Numeracy Framework

Common European Numeracy Framework

- Content
  - Domains (as in PIAAC, PISA; as in mathematics curricula)
  - Big ideas in Mathematics
- Cognitive processes (higher order skills / 21st century skills)
  - Problem solving, reasoning, modelling, ….
- Affective aspects
  - Attitudes / qualities: self-efficacy, self-confidence, no math anxiety, critical interpretation, …
- Contexts / Themes /Life
  - Work, daily-life, in house, in society, public domain (politics, media), private domain (shopping, economic domain (money, rent & mortgage, …)

Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT
bfi
UNIVERSITAT DE BARCELONA
UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

Situational demands

Aspects of numeracy

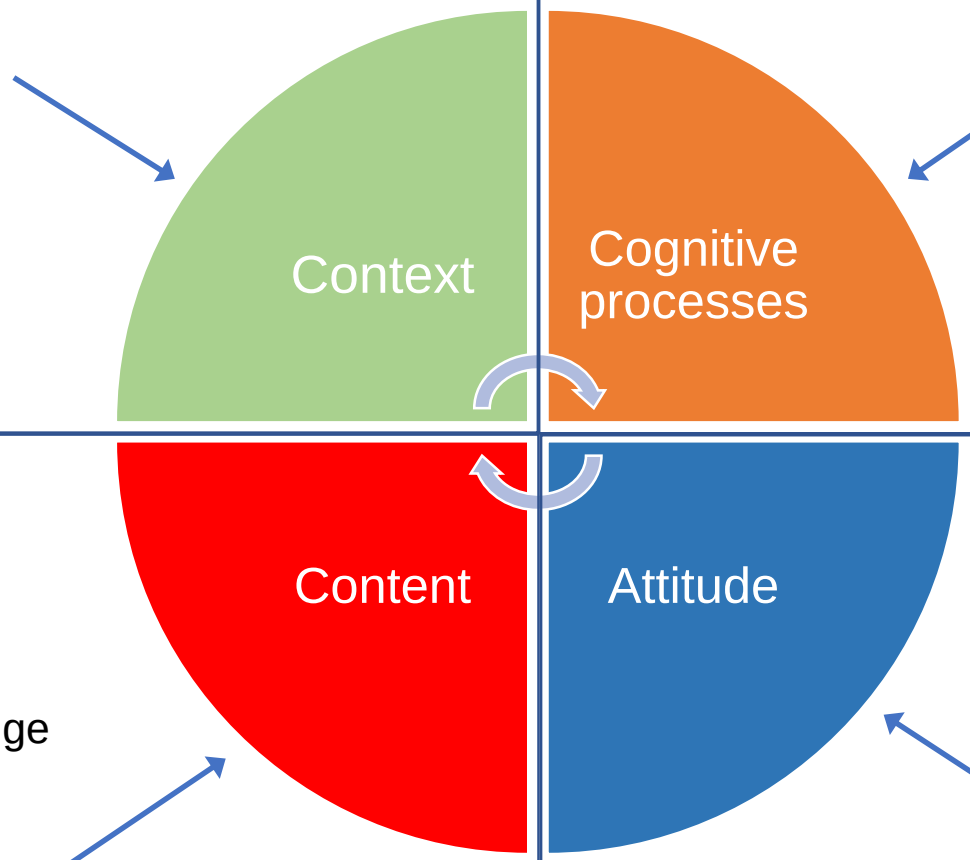
Individual competences

Context

Everyday life
Work-related
Citizenship
Further learning
Financies
Health and care
Recreation

Higher order skills

Managing situations
Analyzing situations
Processing information
Reasoning
Mathematizing
Problem solving
Critical thinking



Knowledge and skills

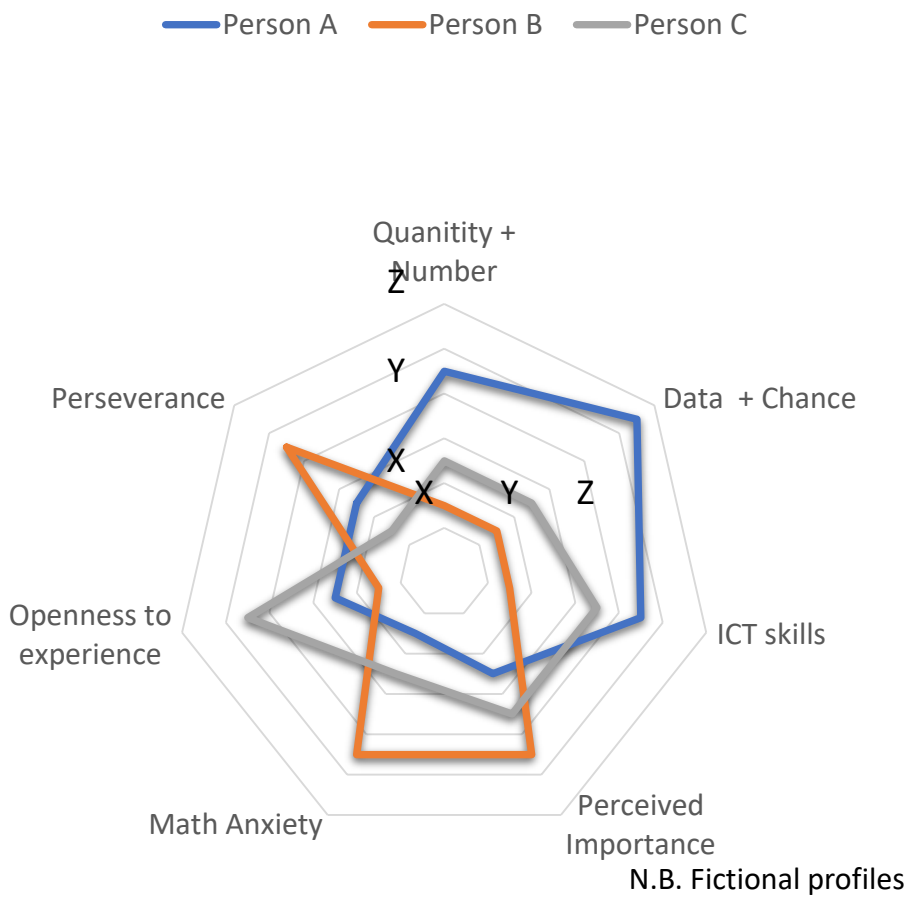
Quantity and number
Dimension and Shape
Pattern, relationships and change
Data and chance
Using a calculator
Using spreadsheets
Using digital skills

Attitude

Self-confidence
Affection
Beliefs
Cooperation
Flexibility
Math anxiety
Learning difficulties

Individual multidimensional profiles

CENF
Common European Numeracy Framework



N.B. Fictional profiles

Content

- Quantity + Number
- Space + Shape
- Relationship + Change
- Data + Chance

Other Skills

- ICT skills

Attitude

- Enjoyment
- Perceived importance
- Intrinsic value
- Usefulness
- Confidence in learning
- Math Anxiety

Personality

- Openness to experience
- Conscientiousness
- Perseverance

For each dimension there should be measuring tools: tests, observations, portfolio proofs, self-evaluations, ….

Co-funded by the Erasmus+ Programme of the European Union

Tasks at 6 levels (first cycle of PIAAC)

Achievement level and score range	Task descriptions
Below Level 1 0 - 175	Tasks at this level require the respondents to carry out simple processes such as counting, sorting, performing basic arithmetic operations with whole numbers or money, or recognizing common spatial representations in concrete, familiar contexts where the mathematical content is explicit with little or no text or distractors.
Level 1 176 - 225	Tasks at this level require the respondent to carry out basic mathematical processes in common, concrete contexts where the mathematical content is explicit with little text and minimal distractors. Tasks usually require one-step or simple processes involving counting, sorting, performing basic arithmetic operations, understanding simple percents such as 50%, and locating and identifying elements of simple or common graphical or spatial representations.
Level 2 226 - 275	Tasks at this level require the respondent to identify and act on mathematical information and ideas embedded in a range of common contexts where the mathematical content is fairly explicit or visual with relatively few distractors. Tasks tend to require the application of two or more steps or processes involving calculation with whole numbers and common decimals, percents and fractions; simple measurement and spatial representation; estimation; and interpretation of relatively simple data and statistics in texts, tables and graphs.
Level 3 276 - 325	Tasks at this level require the respondent to understand mathematical information that may be less explicit, embedded in contexts that are not always familiar and represented in more complex ways. Tasks require several steps and may involve the choice of problem-solving strategies and relevant processes. Tasks tend to require the application of number sense and spatial sense; recognizing and working with mathematical relationships, patterns, and proportions expressed in verbal or numerical form; and interpretation and basic analysis of data and statistics in texts, tables and graphs.
Level 4 326 - 375	Tasks at this level require the respondent to understand a broad range of mathematical information that may be complex, abstract or embedded in unfamiliar contexts. These tasks involve undertaking multiple steps and choosing relevant problem-solving strategies and processes. Tasks tend to require analysis and more complex reasoning about quantities and data; statistics and chance; spatial relationships; and change, proportions and formulas. Tasks at this level may also require understanding arguments or communicating well-reasoned explanations for answers or choices.
Level 5 376 - 500	Tasks at this level require the respondent to understand complex representations and abstract and formal mathematical and statistical ideas, possibly embedded in complex texts. Respondents may have to integrate multiple types of mathematical information where considerable translation or interpretation is required; draw inferences; develop or work with mathematical arguments or models; and justify, evaluate and critically reflect upon solutions or choices.

Compare for instance with CEFR for languages

PROFICIENT USER	C2	Can understand with ease virtually everything heard or read. Can summarise information from different spoken and written sources, reconstructing arguments and accounts in a coherent presentation. Can express him/herself spontaneously, very fluently and precisely, differentiating finer shades of meaning even in more complex situations.
	C1	Can understand a wide range of demanding, longer texts, and recognise implicit meaning. Can express him/herself fluently and spontaneously without much obvious searching for expressions. Can use language flexibly and effectively for social, academic and professional purposes. Can produce clear, well-structured, detailed text on complex subjects, showing controlled use of organisational patterns, connectors and cohesive devices.
INDEPENDENT USER	B2	Can understand the main ideas of complex text on both concrete and abstract topics, including technical discussions in his/her field of specialisation. Can interact with a degree of fluency and spontaneity that makes regular interaction with native speakers quite possible without strain for either party. Can produce clear, detailed text on a wide range of subjects and explain a viewpoint on a topical issue giving the advantages and disadvantages of various options.
	B1	Can understand the main points of clear standard input on familiar matters regularly encountered in work, school, leisure, etc. Can deal with most situations likely to arise whilst travelling in an area where the language is spoken. Can produce simple connected text on topics which are familiar or of personal interest. Can describe experiences and events, dreams, hopes & ambitions and briefly give reasons and explanations for opinions and plans.
BASIC USER	A2	Can understand sentences and frequently used expressions related to areas of most immediate relevance (e.g. very basic personal and family information, shopping, local geography, employment). Can communicate in simple and routine tasks requiring a simple and direct exchange of information on familiar and routine matters. Can describe in simple terms aspects of his/her background, immediate environment and matters in areas of immediate need.
	A1	Can understand and use familiar everyday expressions and very basic phrases aimed at the satisfaction of needs of a concrete type. Can introduce him/herself and others and can ask and answer questions about personal details such as where he/she lives, people he/she knows and things he/she has. Can interact in a simple way provided the other person talks slowly and clearly and is prepared to help.

IO3

erasmus+

1	Can understand and use simple processes such as counting, sorting, performing basic arithmetic operations with whole numbers or money, or recognizing common spatial representations in concrete, familiar contexts where the mathematical content is explicit with little or no text or distractors.
2	Can understand and use basic mathematical processes in common, concrete contexts where the mathematical content is explicit with little text and minimal distractors. Tasks usually require one-step or simple processes involving counting, sorting, performing basic arithmetic operations, understanding simple percents such as 50%, and locating and identifying elements of simple or common graphical or spatial representations.
3	Can identify and act on mathematical information and ideas embedded in a range of common contexts where the mathematical content is fairly explicit or visual with relatively few distractors. Tasks tend to require the application of two or more steps or processes involving calculation with whole numbers and common decimals, percents and fractions; simple measurement and spatial representation; estimation; and interpretation of relatively simple data and statistics in texts, tables and graphs.

4	Can understand and use mathematical information that may be less explicit, embedded in contexts that are not always familiar and represented in more complex ways. Tasks require several steps and may involve the choice of problem-solving strategies and relevant processes. Tasks tend to require the application of number sense and spatial sense; recognizing and working with mathematical relationships, patterns, and proportions expressed in verbal or numerical form; and interpretation and basic analysis of data and statistics in texts, tables and graphs.
5	Can understand and use a broad range of mathematical information that may be complex, abstract or embedded in unfamiliar contexts. These tasks involve undertaking multiple steps and choosing relevant problem-solving strategies and processes. Tasks tend to require analysis and more complex reasoning about quantities and data; statistics and chance; spatial relationships; and change, proportions and formulas. Tasks at this level may also require understanding arguments or communicating well-reasoned explanations for answers or choices.
6	Can understand and use complex representations and abstract and formal mathematical and statistical ideas, possibly embedded in complex texts. Respondents may have to integrate multiple types of mathematical information where considerable translation or interpretation is required; draw inferences; develop or work with mathematical arguments or models; and justify, evaluate and critically reflect upon solutions or choices.

Awareness, cooperation, dissemination

CENF
Common European Numeracy Framework

- European Numeracy Survey 2019
  - Please contribute
- CENF – quarterly newsletter (starting 1 July 2019)
  - Please subscribe
- Research proposals making use of CENF
  - Always welcome and open to cooperation and support
- Policy documents making use of CENF
  - Always welcome and open to cooperation and support

Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT

bfi

UNIVERSITAT DE BARCELONA

UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

CENF
Common European Numeracy Framework

End of presentation

For information, collaboration, and comments, please contact Kees Hoogland

kees.hoogland@hu.nl

Dr. Kees Hoogland | Associate professor of Numeracy and Mathematics in Vocational and Adult Education | Knowledge Center Learning and Innovation | HU University of Applied Sciences Utrecht | Padualaan 97 | 3584 CH Utrecht | The Netherlands | Ph.+316 3410 1701 | Skype: keeshoogland | www.english.hu.nl

Programme manager of Erasmus+ project: Common European Numeracy Framework
Member of the OECD Numeracy Expert Group (2nd cycle of PIAAC)
Editor of Adults Learning Mathematics – International Journal ALM-IJ
Fellow of the International Society for Design and Devolopment in Education
Chair of the Thematic Working Group - Adult Mathematics Education - at CERME 11 (Utrecht, 6-10 February 2019)

Co-funded by the Erasmus+ Programme of the European Union

CENF
Common European Numeracy Framework

Common European Framework of Reference for Language (CEFR)

- CEFR symposium 1992 Report of the Symposium (1992)
- CEFR Companion Volume with New Descriptors 2018

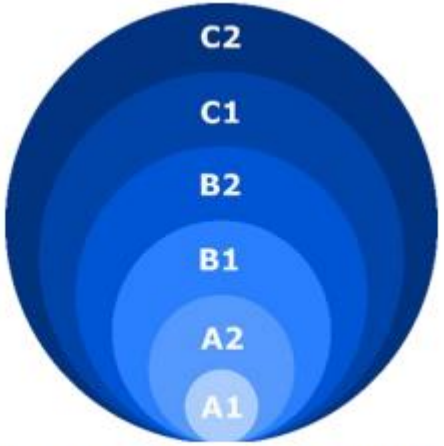
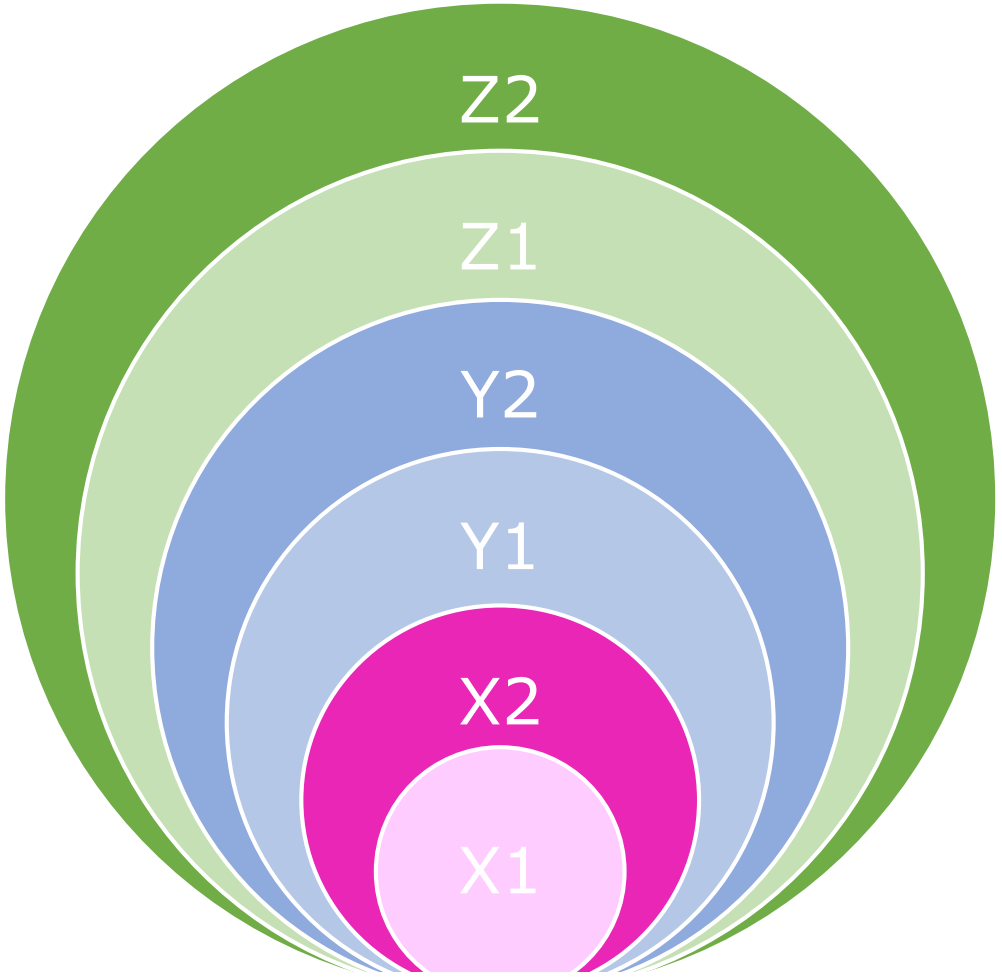


Figure 3 – CEFR Common Reference Levels

Overall levels (= categories ≠ tresholds)

CENF
Common European Numeracy Framework

Z Professional use	Z Specialized societal and work situations	Z Proficient user
Y Citizen use	Y Societal and regular work situations	Y Advanced User
X Personal use	X Daily-life situations	X Starting user



Co-funded by the Erasmus+ Programme of the European Union

UNIVERSITY OF APPLIED SCIENCES UTRECHT

bfi

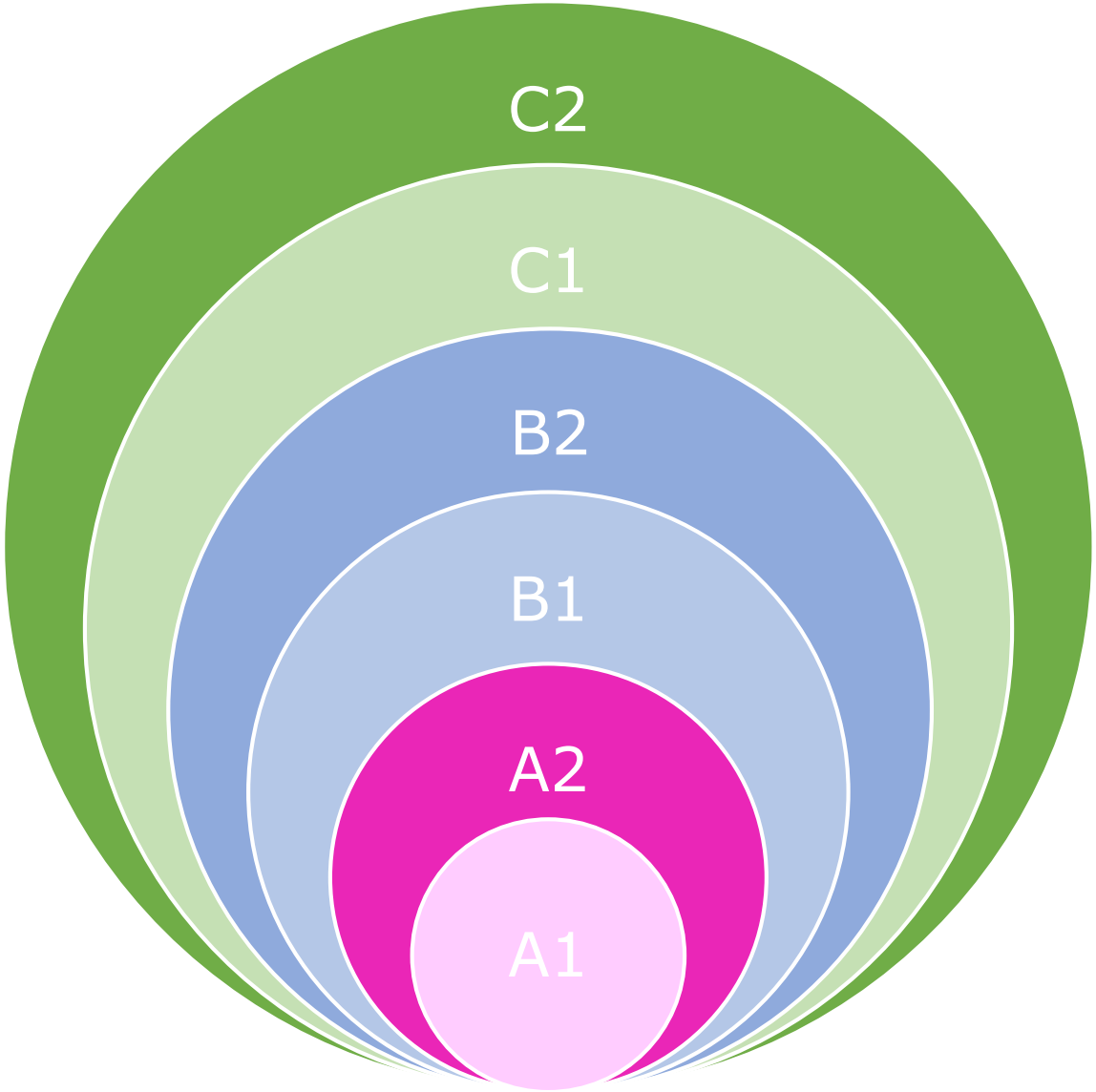
UNIVERSITAT DE BARCELONA

UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

Overal levels (= categories ≠ tresholds)

CENF
Common European Numeracy Framework

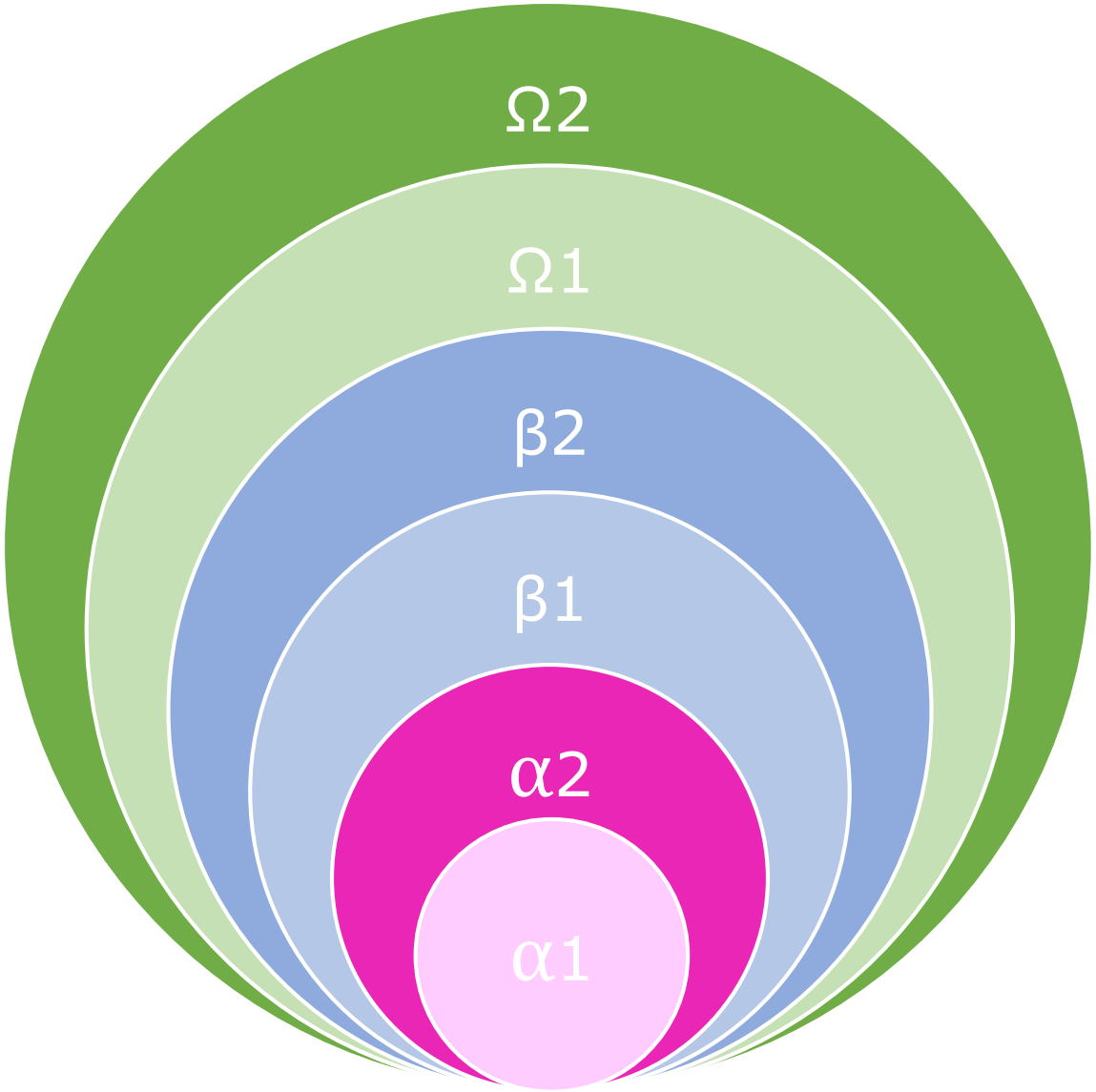
1



Overal levels (= categories ≠ tresholds)

CENF
Common European Numeracy Framework

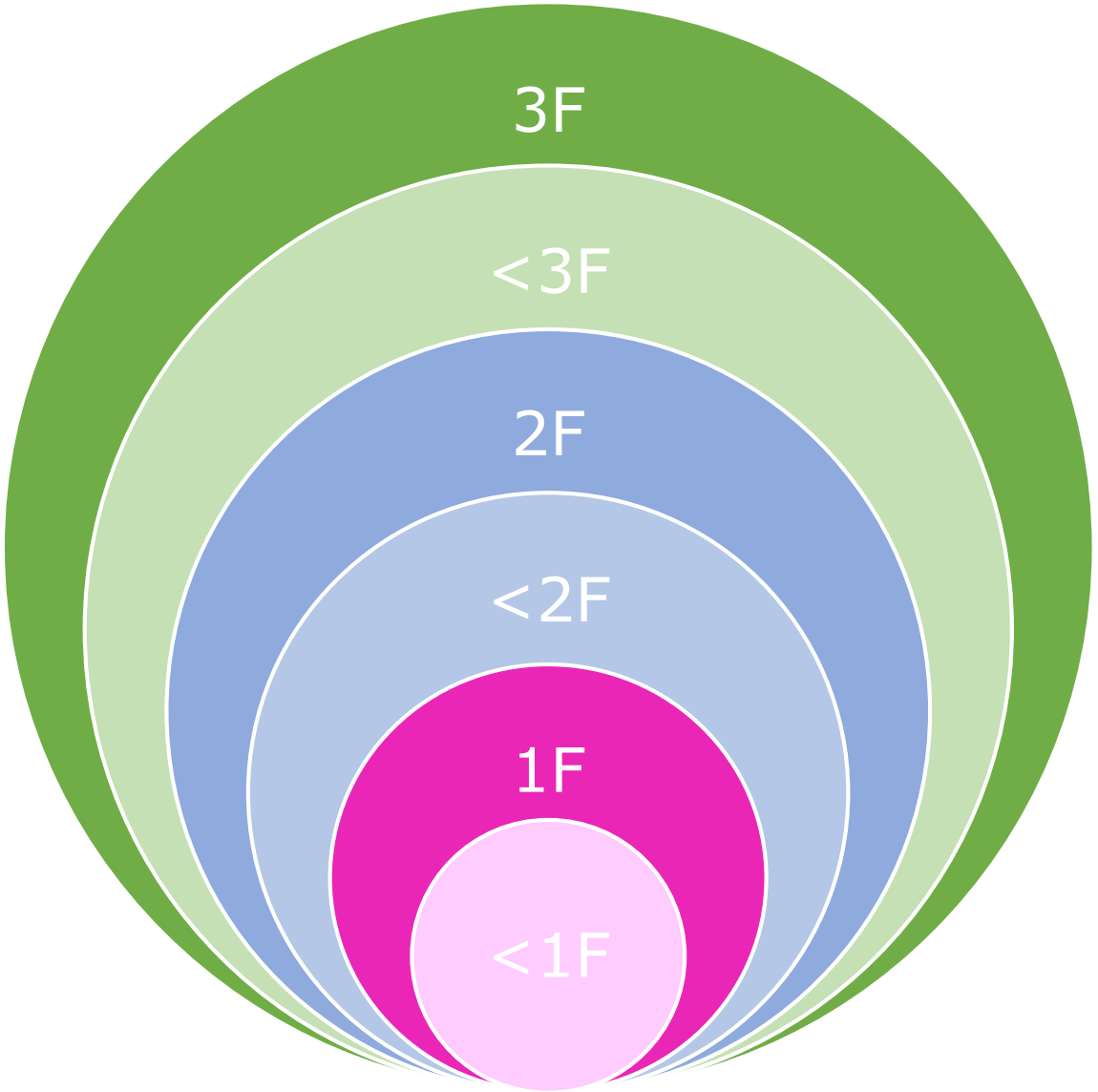
2



Overal levels (= categories ≠ tresholds)

CENF
Common European Numeracy Framework

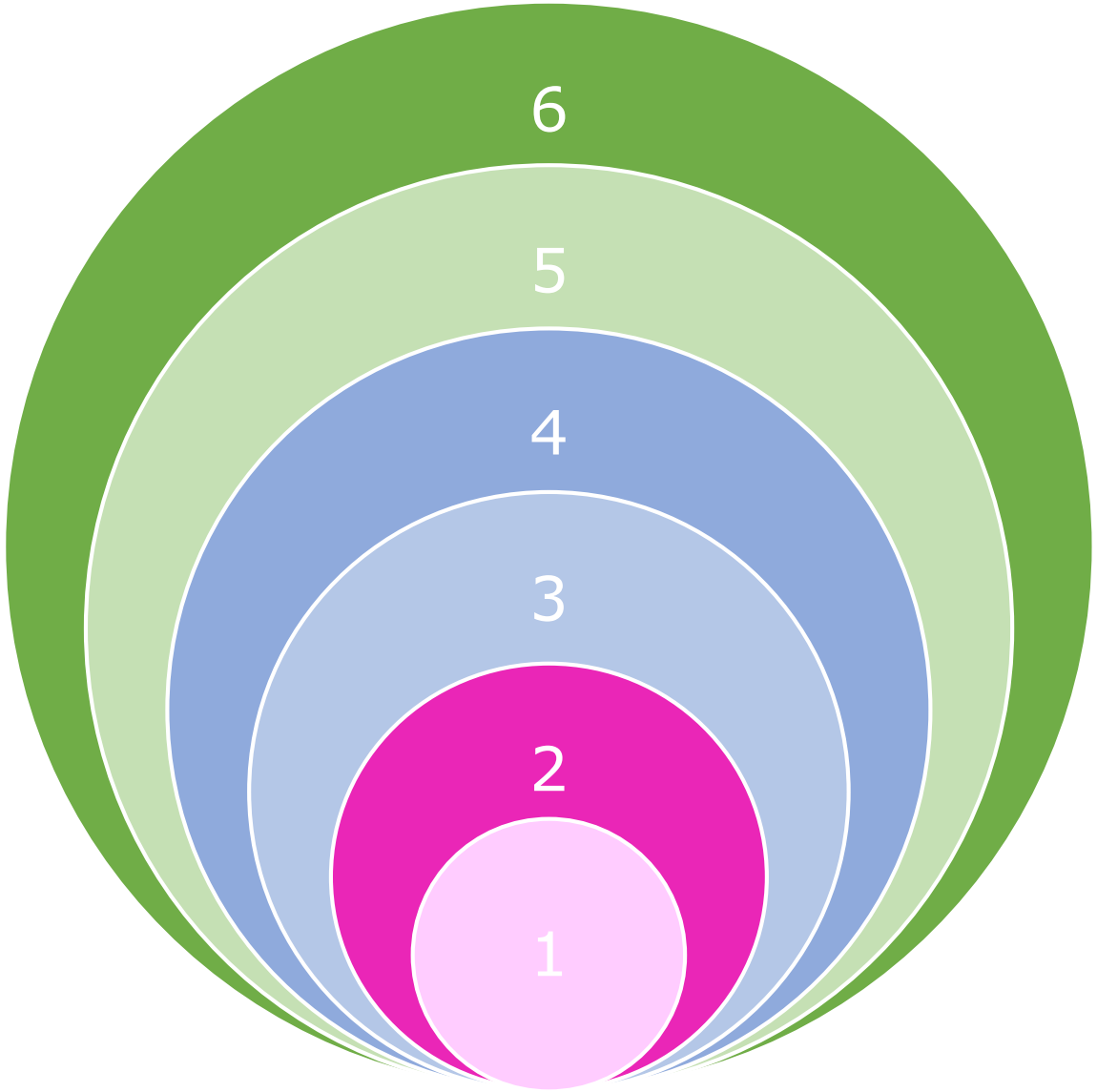
3



Overal levels (= categories ≠ tresholds)

CENF
Common European Numeracy Framework

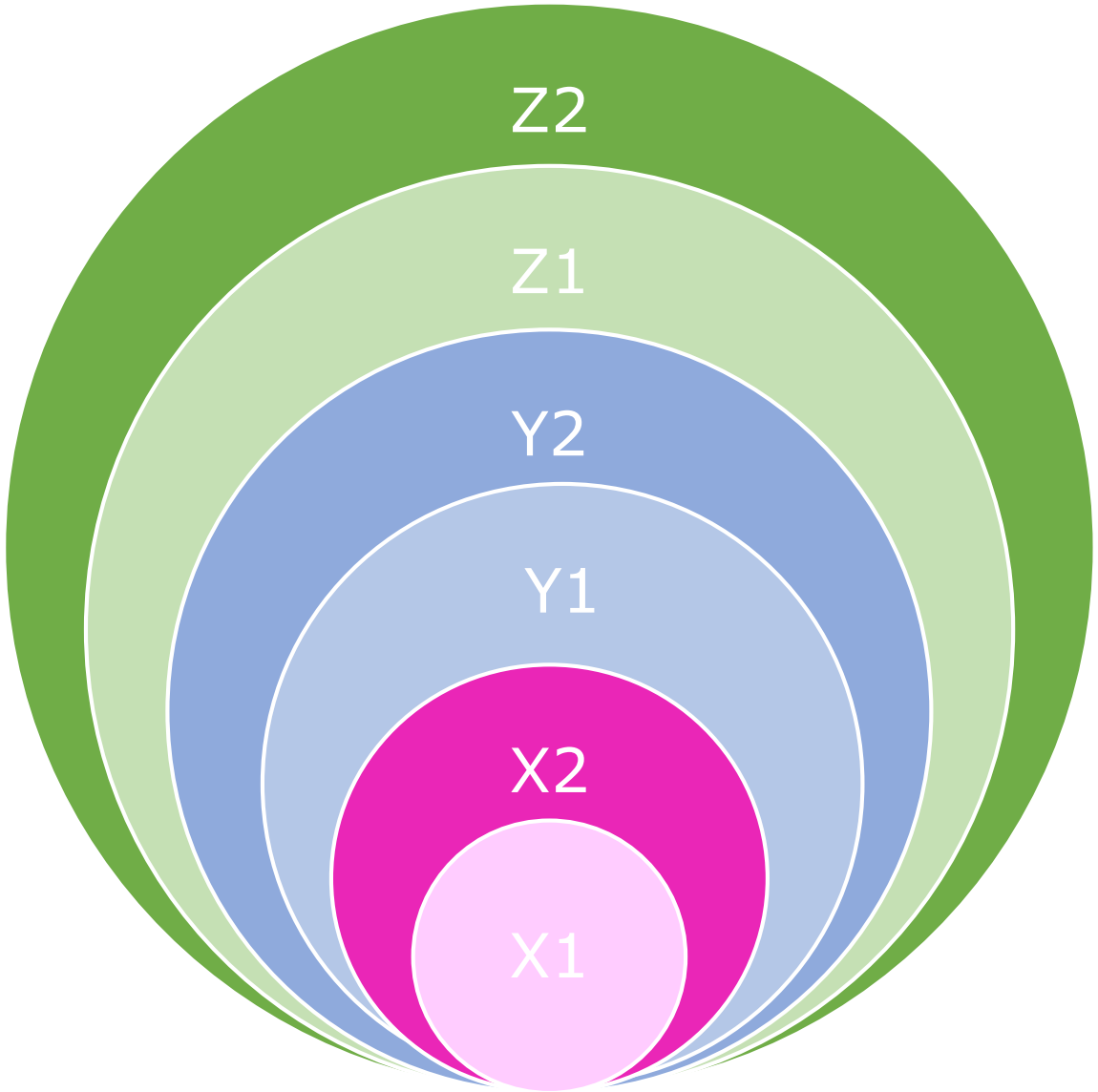
4



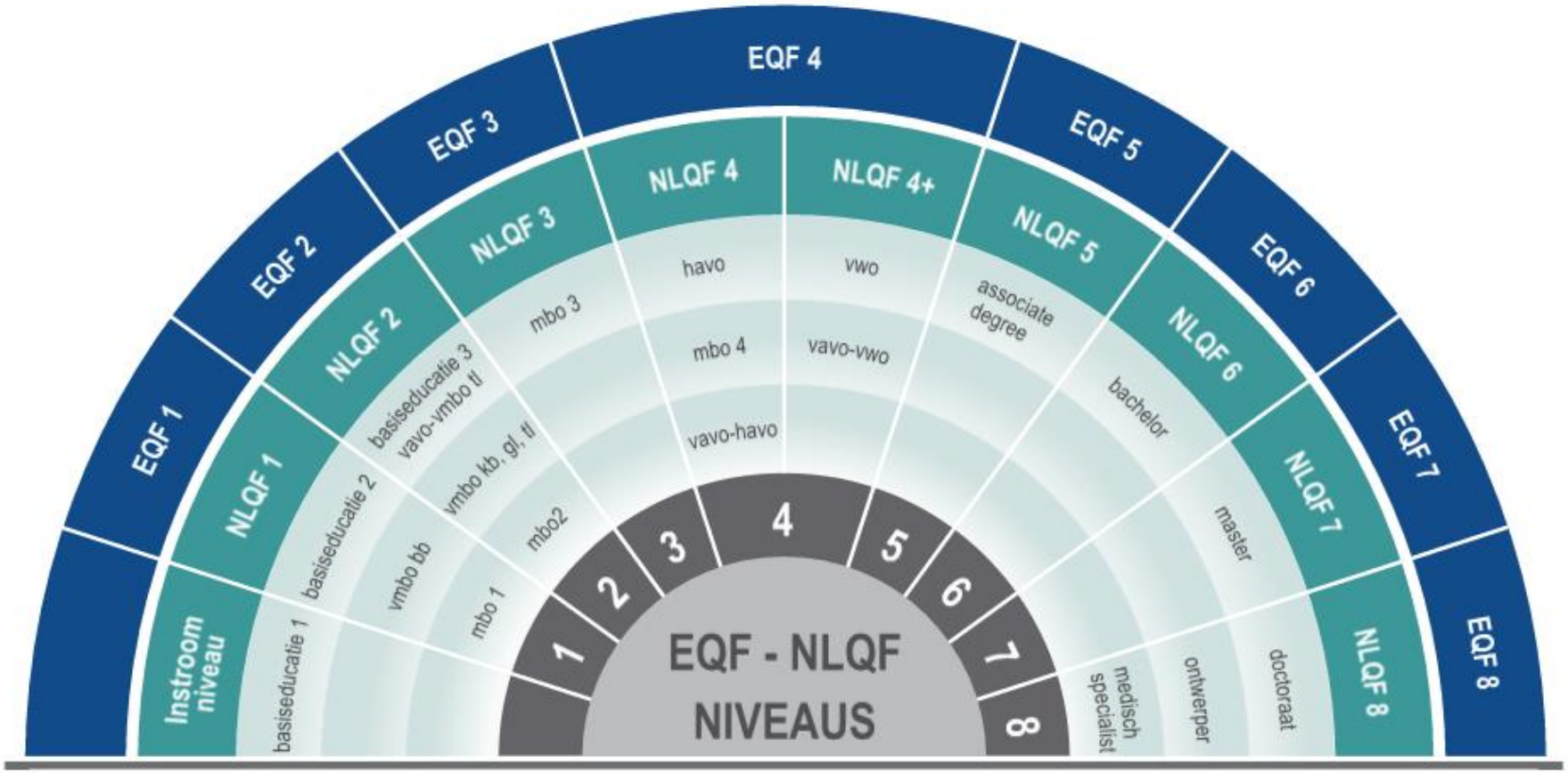
Overal levels (= categories ≠ tresholds)

CENF
Common European Numeracy Framework

5



EQF


EQF 1
EQF 2
EQF 3
EQF 4
EQF 5
EQF 6
EQF 7
EQF 8
NLQF 1
NLQF 2
NLQF 3
NLQF 4
NLQF 4+
NLQF 5
NLQF 6
NLQF 7
NLQF 8
Instroom niveau
basiseducatie 1
basiseducatie 2
vmbo bb
mbo 1
vmbo kb, gl, tl
mbo2
basiseducatie 3
vavo-vmbo tl
mbo 3
havo
mbo 4
vavo-havo
vwo
vavo-vwo
associate degree
bachelor
master
medisch specialist
ontwerper
doctoraat
1
2
3
4
5
6
7
8
EQF - NLQF
NIVEAUS

CENF
Common European Numeracy Framework

Learning and teaching in Adult Education

- Learning in meaningful situations
- Learning by doing
- Learning by communication
- Learning by problem solving
- Learning by taking courses
- Reflection

- Facilitate learning situations
- Listen to the learners' wishes and needs for learning
- Activate prior knowledge
- Coach and support learners
- Encourage learners' initiatives
- Encourage collaborative learning

IO3

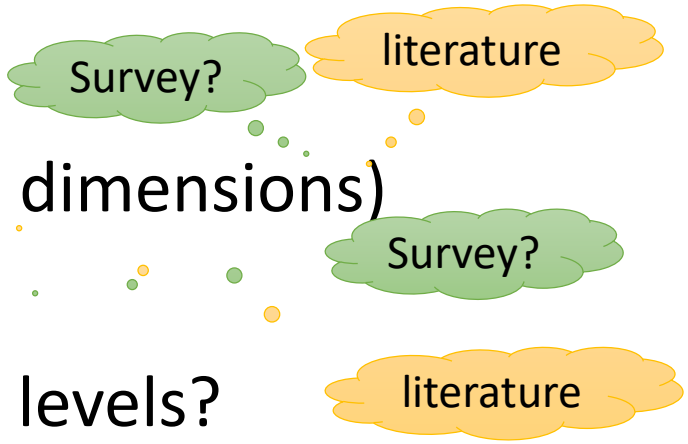
Co-funded by the Erasmus+ Programme of the European Union

erasmus+

CENF
Common European Numeracy Framework

Common European Numeracy Framework

- Are we looking for overall levels, e.g. CERF for languages?
  - X1, X2, Y1, Y2, Z1, Z2
  - K1, K2, L1, L2, M1, M2
- Are we looking for profiles (more scores on more dimensions)
- Are we looking at rubrics to describe behaviour?
- Are we looking at “ideal behaviours” on different levels?

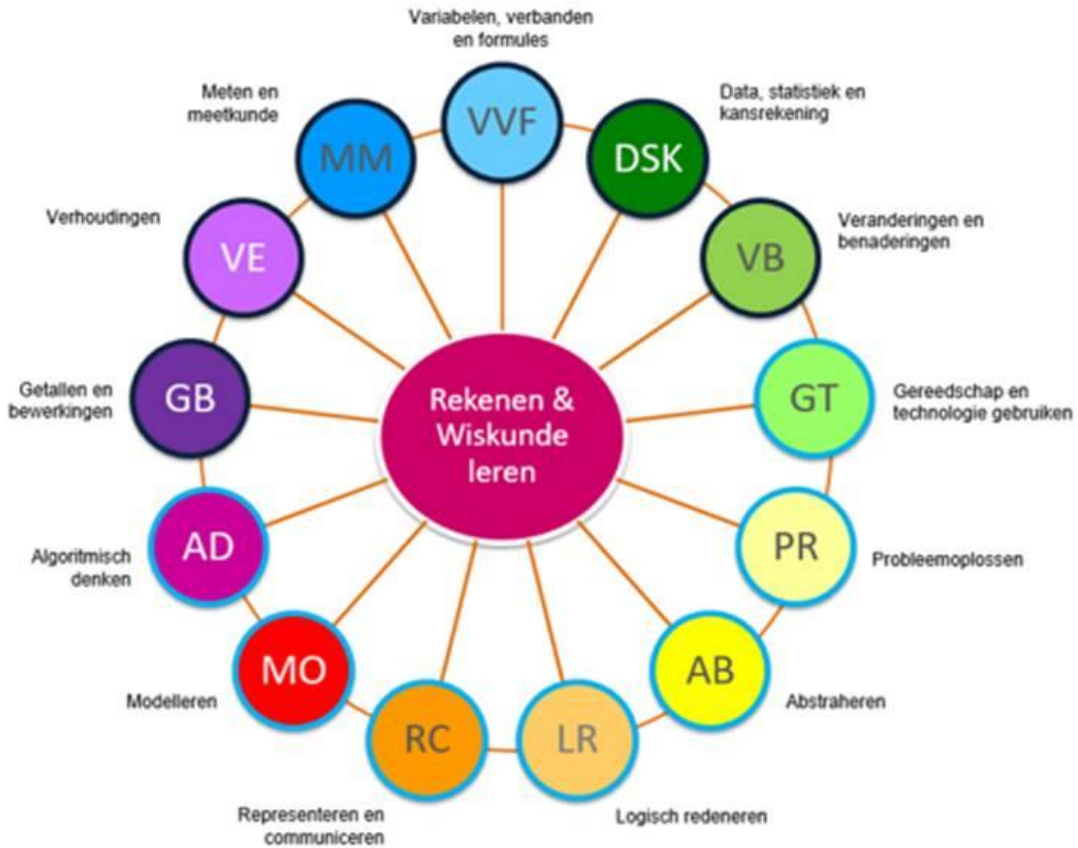


- Is there a fruitful relation between levels and self-realization?

Co-funded by the
Erasmus+ Programme
of the European Union

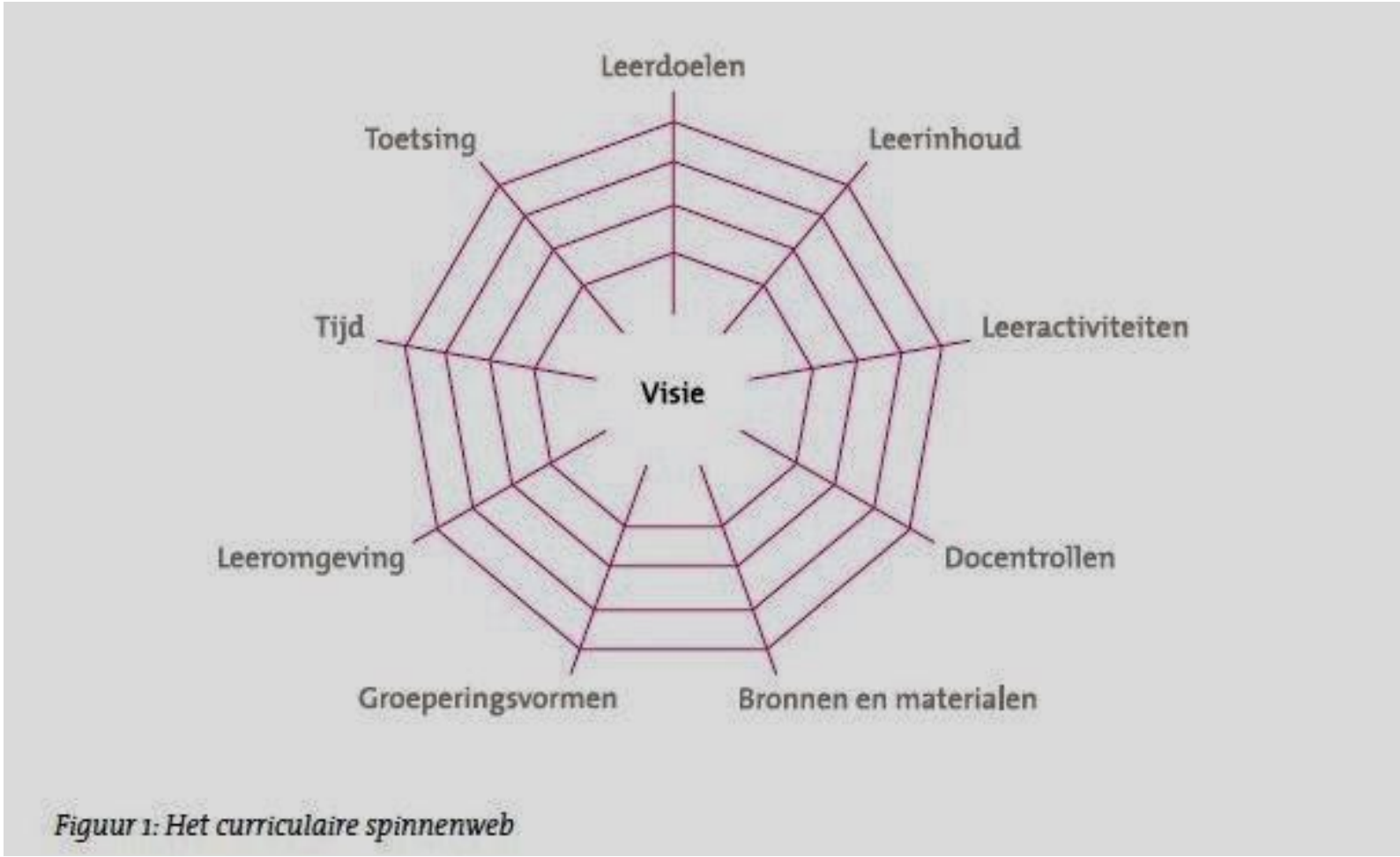
erasmus+

Leergebied Rekenen & Wiskunde



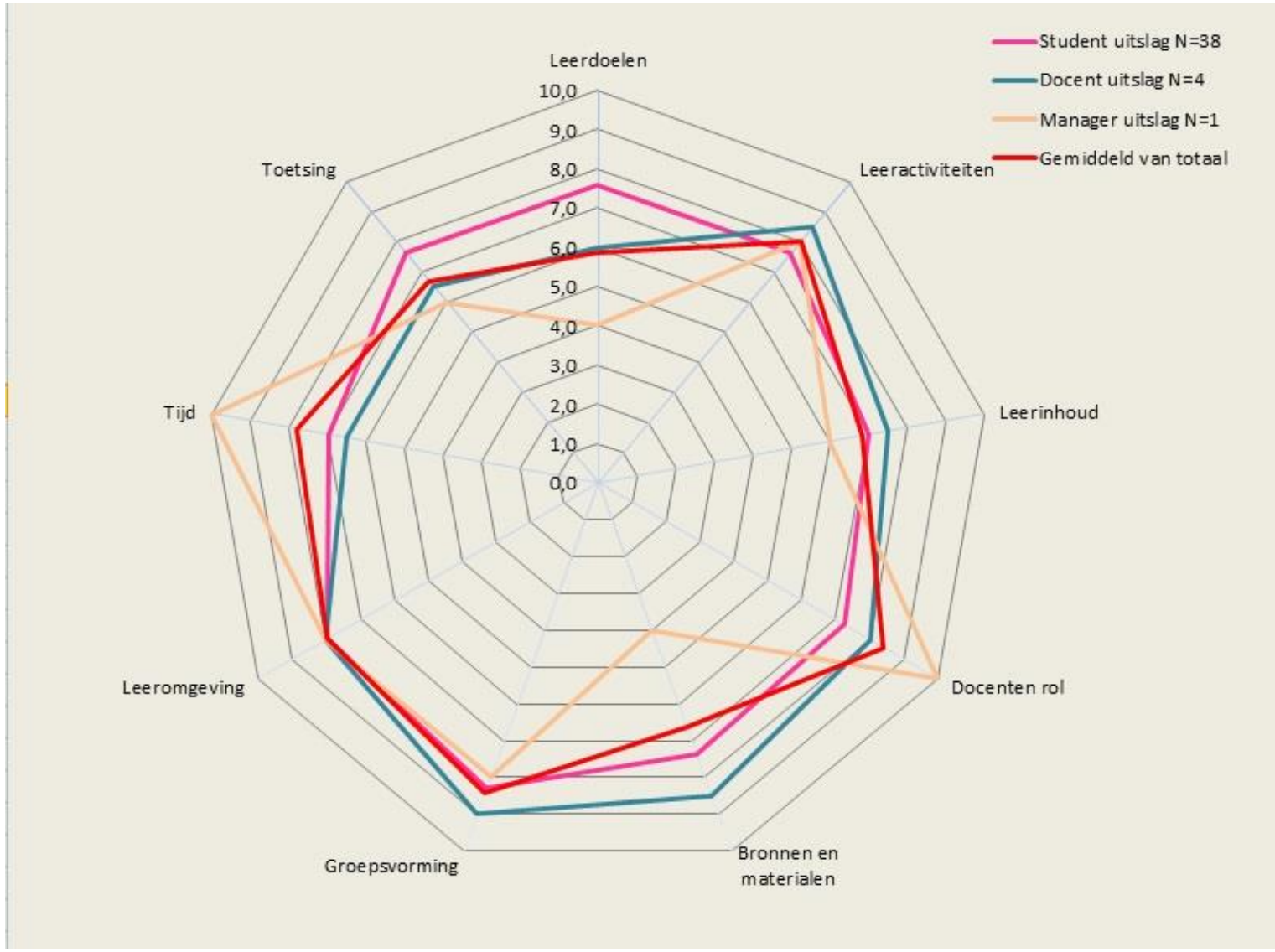
Grote Opdrachten met een donkerblauwe rand betreffen de wiskundige inhouden (WI)

Grote Opdrachten met een lichtblauwe rand betreffen de wiskundige bekwaamheden (WB)



Figuur 1: Het curriculaire spinnenweb


Leerdoelen
Leerinhoud
Leeractiviteiten
Docentenrollen
Bronnen en materialen
Groeperingsvormen
Leeromgeving
Tijd
Toetsing
Visie


Student uitslag N=38
Docent uitslag N=4
Manager uitslag N=1
Gemiddeld van totaal
Leerdoelen
Leeractiviteiten
Leerinhoud
Docenten rol
Bronnen en materialen
Groepsvorming
Leeromgeving
Tijd
Toetsing
10,0
9,0
8,0
7,0
6,0
5,0
4,0
3,0
2,0
1,0
0,0